Numerical Evaluation of Inelastic Mechanisms in a Nitrogen Arcjet

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Introduction

THE electrons in a low-power arcjet thruster are thermally excited through strong interactions with the electric field via ohmic heating and may be out of equilibrium due to low density and small thruster dimensions. Numerical evaluation of such nonequilibrium states is not only important for accurate prediction of thruster performance but also of interest from the viewpoint of fluid dynamics. For a propellant composed of molecular species, the rate of electron energy loss due to inelastic collisions with molecules can be of many orders of magnitude larger than that due to elastic collisions. One common approach taking into account these effects is to multiply the collision frequencies for electron–molecule encounters by the inelastic loss factor δ, but there exist considerable uncertainties in determining the value of δ, which was assumed to be constant in Refs. 1 and 2. In fact, inelastic mechanisms are expected to be strongly nonlinear with respect to electron energy distribution as pointed out in Ref. 3.

This Note attempts to evaluate the degree of the electron–molecule inelastic effects for a nitrogen arcjet by consistently accounting for each energy transfer mechanism, not assuming a constant δ. This is essentially the same approach as the work of Megli et al.[4] in the sense that the δ value can be evaluated from the solution obtained, but emphasis is placed on the detailed description of the different mechanisms involved and on the determination of the relative importance among them.

Formulation

The governing equations that describe the flow in an arcjet are the Navier–Stokes equations extended to include equations for the conservation of each gas species and the electron energy. A total of eight chemical reactions associated with five species (N_2, N, N_2^+, N^+, and e^-) is considered here. Coupled with Maxwell's equations, they are discretized and solved numerically in a finite volume fashion. The detailed descriptions of the model and numerical procedures are found in Refs. 5 and 6. The effects of the energy loss due to elastic and inelastic collisions appear in the electron energy conservation equation and must be modeled. In Ref. 7, Appleton and Bray constructed an electron energy equation for monatomic species that includes elastic/inelastic interaction terms by assuming Maxwellian velocity distribution for the electron energy. In the present model, we extend their formulation to include some relevant mechanisms that are important for diatomic species such as nitrogen. Aside from the elastic collisions between electrons and heavy particles, the electrons lose energy by the electron-impact dissociation/ionization reactions. This is a main inelastic loss mechanism for monatomic gases, and additional effects must be taken into account for a plasma composed of molecular species. It is well known that the low-energy electron impact vibrational excitation for the electronic ground state of N_2 is extremely strong due to the resonant mechanism, so that the energy source term representing this effect must be modeled. The energy exchange rate between the electron and the rotational modes may become important on some occasions as well. Consistently incorporating these mechanisms, we can now describe the explicit expression of the electron energy dissipation term S_e as

$$S_e = Q_{T-e} + Q_{R-e} - Q_{e-V} + Q_D^e + Q_I^e \tag{1}$$

The rate of the translational energy transfer between heavy particles and electrons is given by[7]

$$Q_{T-e} = \sum_{s=1}^{4} n_e \frac{2m_e}{n_s} \nu_{e,s} \frac{3}{2} k(T - T_e) \tag{2}$$

where $\nu_{e,s}$ is the collision frequency of electron-s species encounters and can be evaluated as found in Ref. 8. The energy transfer between the rotational mode and the electron energy mode should also be taken into consideration as Q_{R-e}. This is because electrons interact with molecular multipoles produced by deformation of the electron cloud in the formation of molecular bonds.[9] The energy transfer rate is written as

$$Q_{R-e} = \sum_{s=\text{mol}} n_e \frac{2m_e}{m_s} g_{\text{rot},s} \nu_{e,s} \frac{3}{2} k(T - T_e) \tag{3}$$

The factor $g_{\text{rot},s}$ is the ratio of Q_{R-e} to Q_{T-e} for molecular species. In the present work, we assume $g_{\text{rot},N_2} = g_{\text{rot},N_2^+} = 10$ (Ref. 10). As for the electron–vibration coupling, Lee[11] developed an approximate rate equation and a relaxation time by solving the master equation for vibrational transitions, including the effects of multiple-level transitions from and to nonground states. Because the obtained relaxation time agrees well with experimental data, this model appears to be quite reliable and is employed here. This can be expressed as

$$Q_{e-V} = \rho_{N_2} \left\{ \left[e^{\text{eq}}_{\text{vib},N_2}(T_e) - e_{\text{vib},N_2}(T_{\text{vib}}) \right] / \tau_e \right\} \tag{4}$$

The relaxation time τ_e is evaluated as a function of electron temperature and electron pressure, and expressed by the curve-fit formula found in Ref. 11. Finally, in both electron-impact dissociation and ionization, dissociation/ionization energy is mainly extracted from the electron energy,[12] so that the loss of the electron energy due to these reactions may be written as

$$Q_D^e = E_{D,N_2} \dot{w}^e_{N_2}, \qquad Q_I^e = E_{I,N} \dot{w}^e_N \tag{5}$$

where $\dot{w}^e_{N_2}$ and $\dot{w}^e_N$ are the mass production rate of N_2 and N due to electron-impact dissociation and ionization, respectively.

Results and Discussion

The results are presented for a case at the mass flow rate of 45.4 mg/s and the total current of 6 A to reproduce the experimental data in Ref. 13. The constrictor length and diameter and the nozzle area ratio are 0.25 mm, 0.64 mm, and 225, respectively.

As stated earlier, a classical way to take into account a series of inelastic collisions between the electron and molecular species is to introduce the inelastic loss factor δ and write the average energy loss rate as[14]

$$\sum_{s=1}^{4} n_e \frac{2m_e}{m_s} (\delta_s \nu_{e,s}) \frac{3}{2} k(T - T_e)$$

where $\delta_s = 1$ for atomic species ($s = $ N and N^+). In the present formulation, using the flow properties obtained from the computation, we can explicitly evaluate the value of δ from the following relations:

$$\sum_{s=1}^{4} n_e \frac{2m_e}{m_s} (\delta_s \nu_{e,s}) \frac{3}{2} k(T - T_e) = Q_{T-e} + Q_{R-e} - Q_{e-V} \tag{6}$$

where $\delta_{N_2} = \delta_{N_2^+} = \delta$ is assumed. Note that the electron energy loss due to electron impact dissociation/ionization is not included in the right-hand side of Eq. (6). This is essentially the same definition as those of Refs. 1 and 2.

Shown in Figs. 1 and 2 are the distributions of electron temperature and calculated δ in the constrictor region, respectively. A

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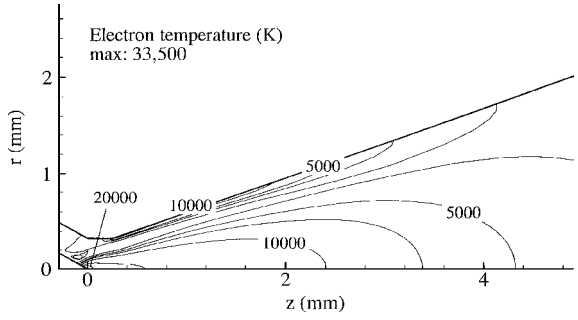

Fig. 1 Electron temperature contours.

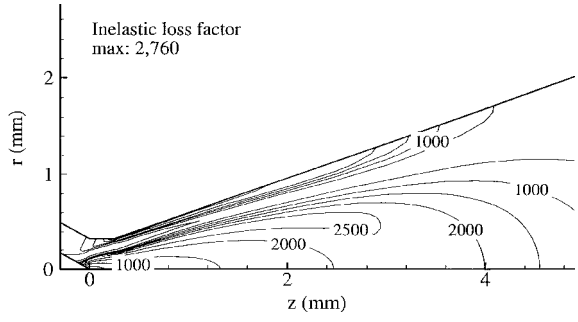

Fig. 2 Inelastic loss factor distribution.

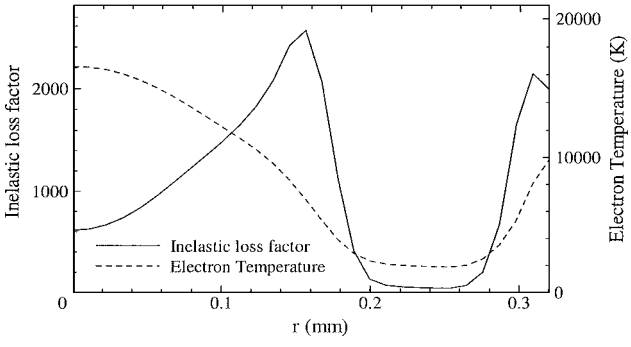

Fig. 3 Profile of δ at the constrictor exit.

maximum of $\delta \approx 3000$ is noted at the location of $z = 0.45$ mm and $r = 0.22$ mm.

It is ensured that δ strongly depends on the energy distribution and is mainly a function of the electron temperature. However, as found by comparing Figs. 1 and 2, the higher T_e does not always correspond to the higher δ. This is because the thermal relaxation time for the vibration–electron energy exchange process [τ_e in Eq. (4)] shows a minimum around $T_e \approx 7000$ K in the present model.[11] Examining Fig. 3, we can confirm that δ becomes large toward this temperature. Because the degree of vibration–electron coupling is much stronger than that of other inelastic loss processes for N_2, this term plays a critical role in evaluating δ. Also note that the computed δ is of the order of 10^3 as predicted by Martinez-Sanchez and Miller.[3]

Summary

A numerical model was developed to reproduce the nonequilibrium flowfield in a nitrogen arcjet thruster. The mechanisms for inelastic collisions between electrons and molecules were investigated in detail by considering the rotation–electron and the vibration–electron couplings. A computation was performed for a low-power operating condition, and it was shown that the energy transfers due to inelastic collisions significantly depend on the electron energy distribution, mainly owing to strong nonlinearity of the vibrational relaxation process via electron impact.

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